Inconel 718 : SN curve modeling

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Materials reliability

Question

How many flies, an aircraft engine disk can do ?

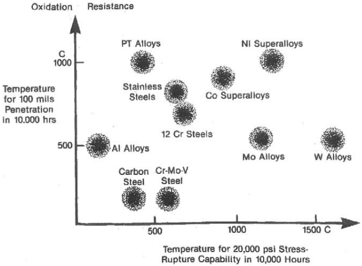
Area of our study

Study of Inco718 a superalloy used by Snecma

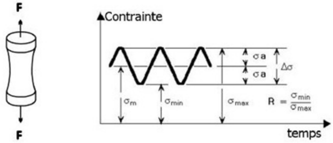
Inconel 718

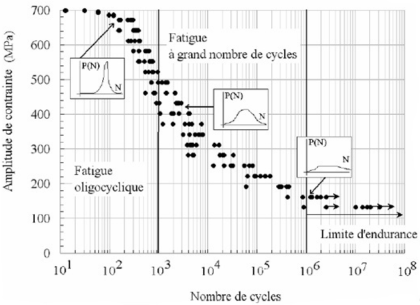
Elements (%)	Ni	C	Cr	Fe	Nb	Mo	Ti	Al
mini	Base	0.02	17	15	4.75	2.80	0.75	0.3
maxi	Base	0.08	21	21	5.50	3.30	1.15	0.7

Table: Alloy 718



Fatigue Tests





Usual models

Lifetime

$$Y = \ln(N) = \mu(S) + \sigma\,\epsilon$$

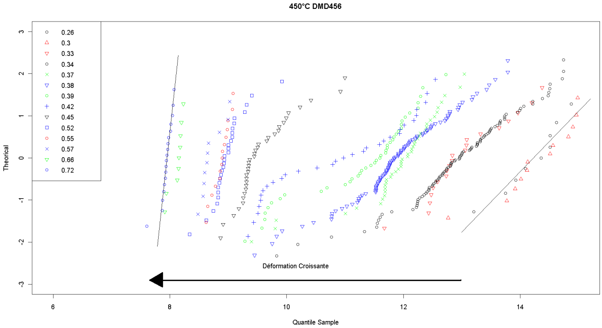
Accelerated models

- Wöhler : $\mu(S) = a + b\,S$ (1870)
- Basquin : $\mu(S) = a + b\,\ln(S)$ (1912)
- $\ldots$
- Pascual - Meeker : $\mu(S) = a + b\,\ln(S - E)$ (1999)

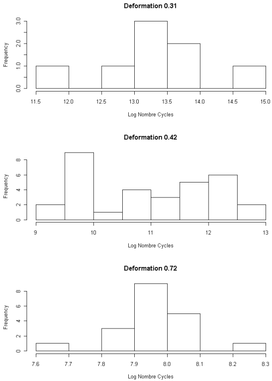
Our data

Our data are bi-modal. Those model with an unique distribution don't give accurate fitting

Quantile Plot



Double SN curve



Initiation and propagation

$$N = N_a + N_p$$

Mixture and classification

$$N^p \sim \phi(\theta_{propagation}) \quad \text{et} \quad N^a \sim \phi(\theta_{initiation})$$

and

$$N = \begin{cases} N^p, & \text{if} \quad Z = 1 \quad \text{with} \quad \pi_1 = \mathbf{P}(Z = 1)\,; \\ N^a + N^p, & \text{if} \quad Z = 2 \quad \text{with} \quad \pi_2 = \mathbf{P}(Z = 2)\,; \end{cases}$$

We observe N but the conditions of the initiation Z are unkonwn
π_1 : Probability of an initiation at the first cycle.

Density

observed data

$$\pi_1(S_i) \cdot \phi_{N^p}(N_i) + \pi_2(S_i) \cdot \phi_{N^p+N^a}(N_i)$$

Accelerated test

$$N^p \sim \mathcal{LN}(\mu_p(S)\,;\,\sigma_p^2)$$
$$\mu_p(S) = a_p + b_p\,S$$
$$\theta_p = (a_p, b_p, \sigma_p^2)$$

LogLikelihood

LogLikelihood

$$L(N \mid S, \theta) = \prod_{i=1}^{n} \pi_1(S_i) \cdot \phi_{N^p}(N_i) + \pi_2(S_i) \cdot \phi_{N^p+N^a}(N_i)$$

with $\pi_1(S) = \frac{e^{(\alpha+\beta*S)}}{1+e^{\alpha+\beta*S}}$

Identifiability

Sufficient condition : $\sigma_a > \sigma_p$

Maximisation

We will use the EM algorithm to maximise this quantity

Convolution product

Expression

$$\phi_{N^a+N^p}(N_i) = \int_0^{N_i} \phi_{N^a}(x)\phi_{N^p}(N_i - x)dx$$

Approximation with a density

- LogNormal, Log Skew-Normal...

Numerical evaluation

- Monte-Carlo

EM Algorithm

0) Initialisation of $Z^{(0)}$: first clustering of the data, $j = 1, \ldots, n$, we calculate π_1^0 and θ_p^0, θ_i^0 ;

1) Then using the $Z^{(\ell)}$, we iteratively calculate the $\pi_1^{(\ell)}$ and $\theta_p^{(\ell)}, \theta_i^{(\ell)}$:

Expectation step

- For all data :

$$p_{j1}^{(\ell+1)} = \mathbf{P}\left(Z_j = 1 | N = n_j; S_j, \theta^{(\ell)}\right)$$

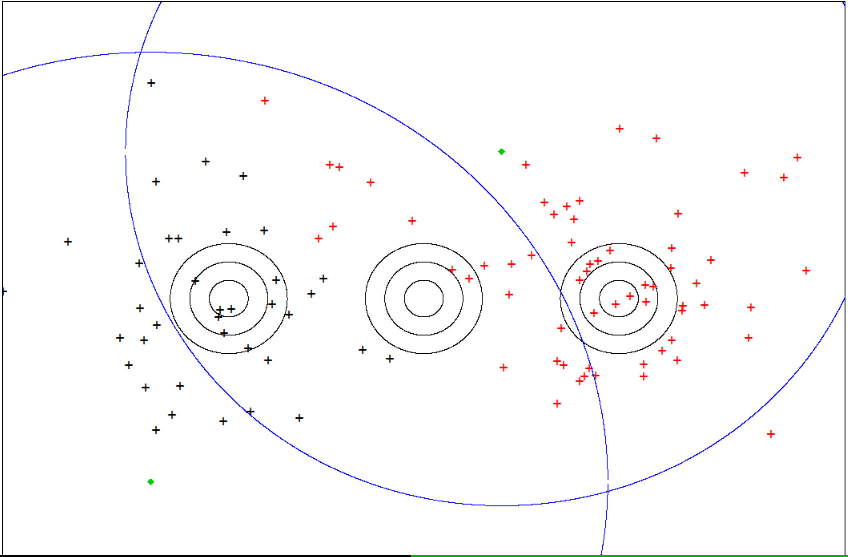
EM Algorithm

Maximisation step

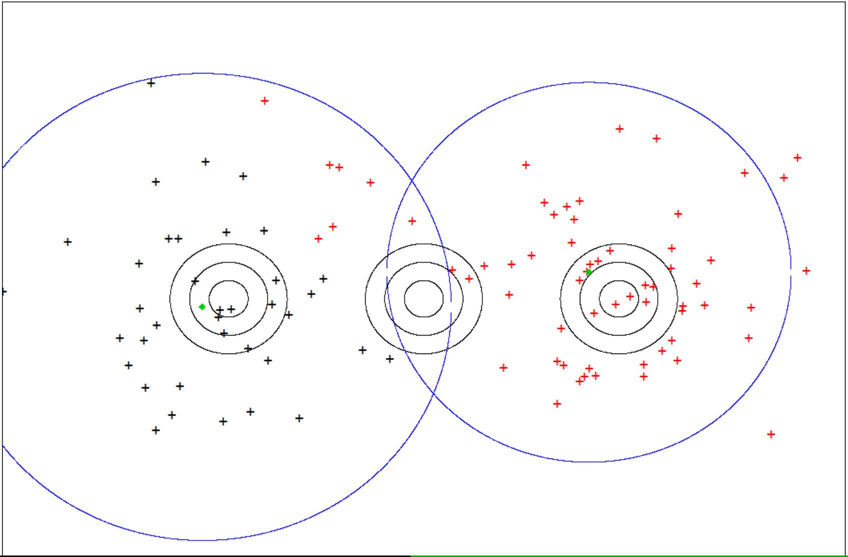
We have to maximise the following expression :

$$\arg\max_{\theta} E(\log L^S(Y\,|\theta)\,|N,\theta^{(\ell)}))$$

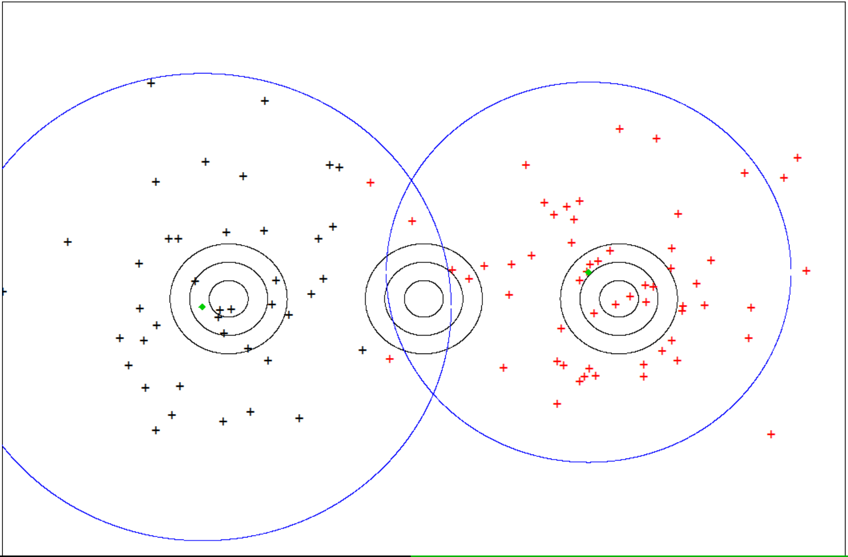
EM Algorithm



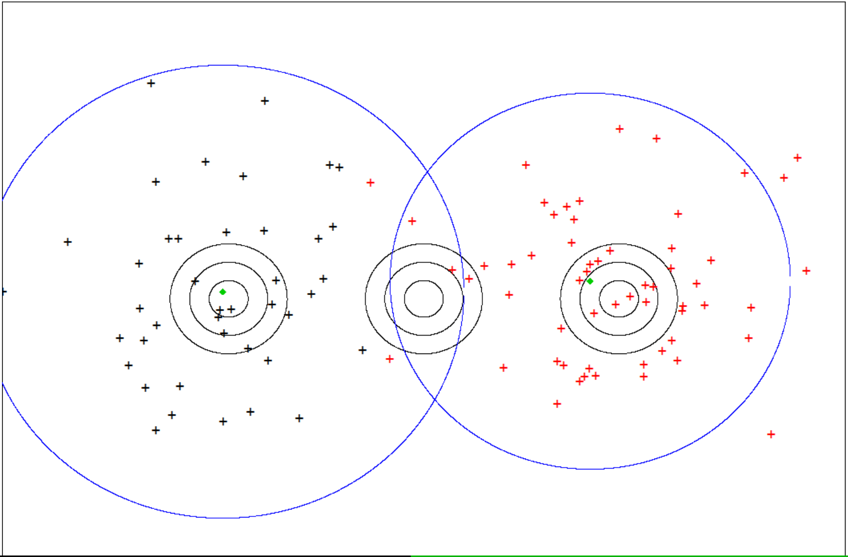
EM Algorithm



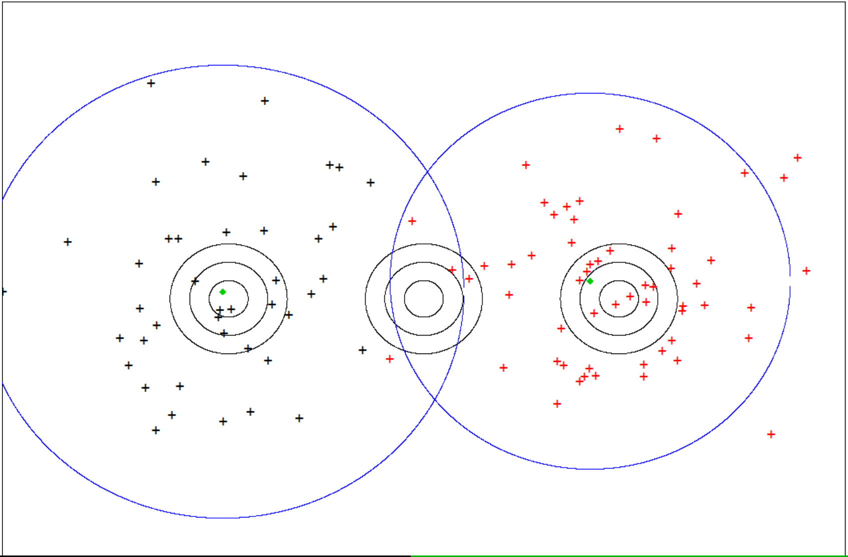
EM Algorithm



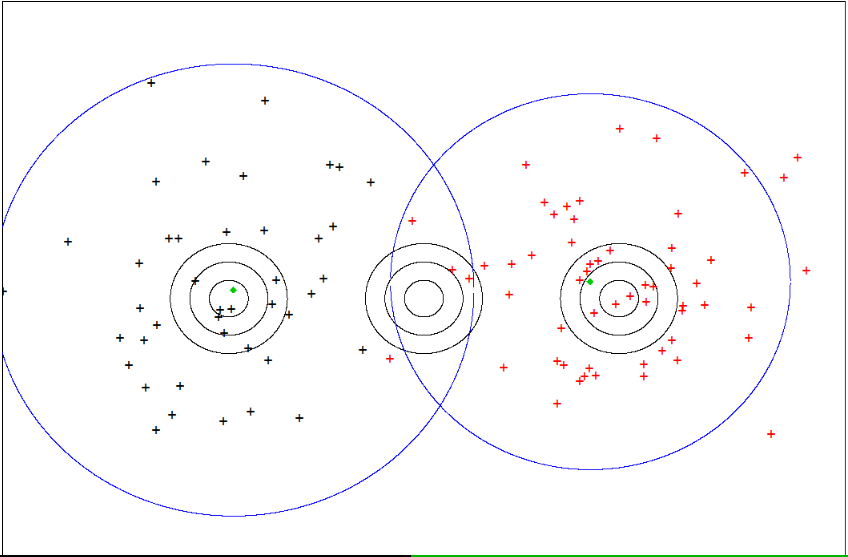
EM Algorithm



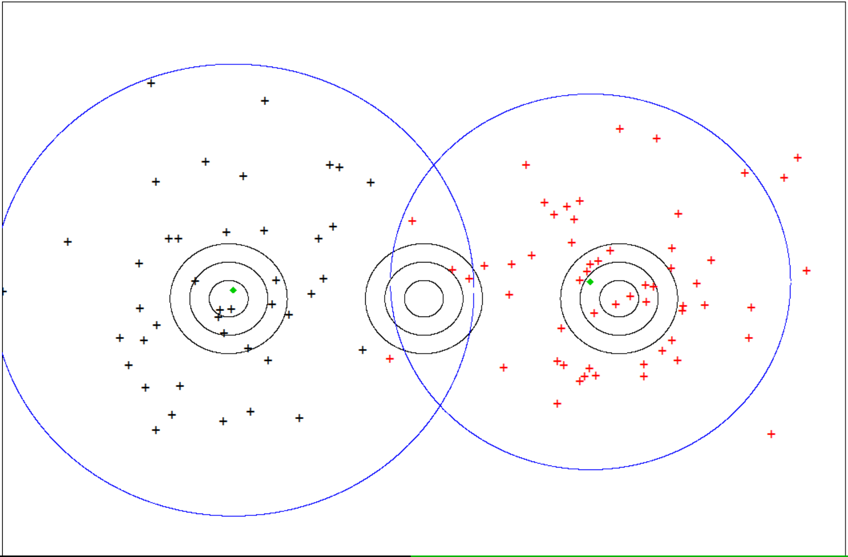
EM Algorithm



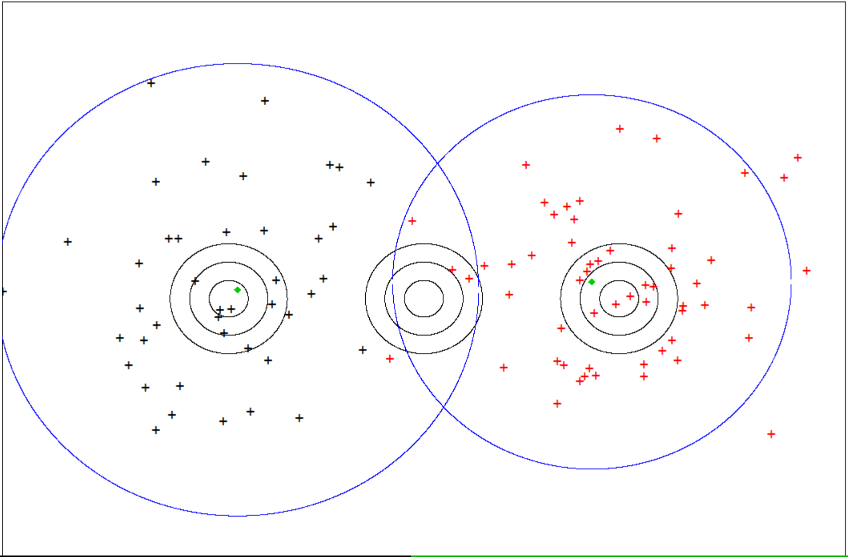
EM Algorithm



EM Algorithm



EM Algorithm



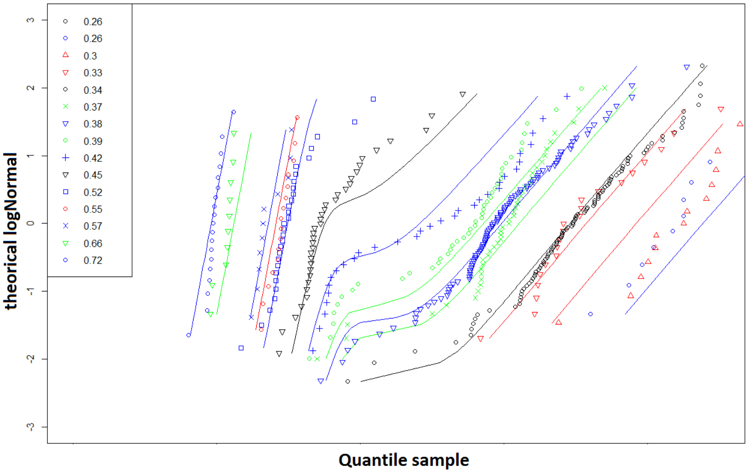
Results - Simulated data with 600 points

parameters	α	β
simulation parameters	15	-35
mean over 250 simulations	17.5	-40.9
standard deviation over 250 simulations	4.9	11

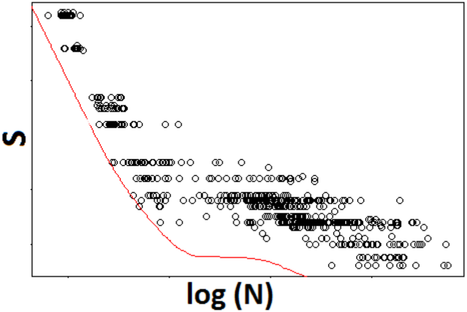
parameters	α_i	β_i	σ_i
simulation parameters	22	-10	.2
mean over 250 simulations	21.9	-9.99	.20
standard deviation over 250 simulations	.11	.16	.01

parameters	α_p	β_p	σ_p
simulation parameters	33	-30	1
mean over 250 simulations	33.1	-30.3	.99
standard deviation over 250 simulations	.54	1.71	.08

Results - QQplot

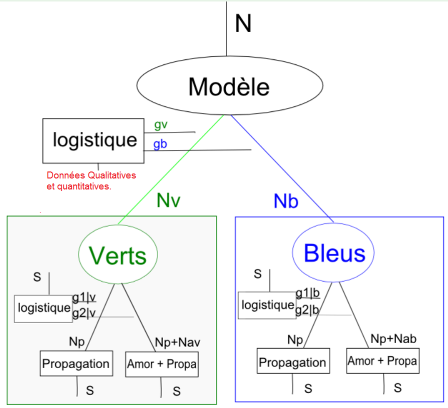


Results -Quantile



Back to the future

Production data



Discussion

That's All folks